SOLVED PROBLEMS IN PROBABILITY

CIE Examination Review Course

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THEOREMS IN PROBABILITY

Theorem #1: If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, then the two operations can be performed in $\boldsymbol{n_1\, n_2}$ ways.--> Multiplication Rule.

Theorem #2: If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of operations can be performed in $\boldsymbol{n_1\, n_2 \ldots n_k}$ ways.

Theorem #3: The number of permutations of $\boldsymbol{n}$ distinct objects is $\boldsymbol{n!}$.

Theorem #4: The number of permutations of $\boldsymbol{n}$ distinct objects taken $\boldsymbol{r}$ at a time is $\boldsymbol{{}_nP_r = n! / (n - r)!}$.

Theorem #5: The number of permutations of $\boldsymbol{n}$ distinct objects arranged in a circle is $\boldsymbol{(n - 1)!}$.

Theorem #6: The number of distinct permutations of n things of which $\boldsymbol{n_1}$ is of one kind, $\boldsymbol{n_2}$ is of a second kind,...., $\boldsymbol{n_k}$ is of a k^{th} kind is $\boldsymbol{n! / n_1!n_2!\ldots n_k!}$.

Theorem #7: The number of ways of partitioning a set of n objects into r cells with $\boldsymbol{n_1}$ in the first cell, $\boldsymbol{n_2}$ in the second cell, and so forth, is $\boldsymbol{n! / n_1!n_2!\ldots n_r!}$.

Theorem #8: The number of combinations of n distinct objects taken r at a time is $\boldsymbol{n! / r!(n - r)!}$.

Theorem #9: If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A, is $\boldsymbol{P(A) = n / N}$.

Theorem #10: If A and B are any two events, then

$$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}.$$

Theorem #11: For 3 events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Theorem #12: If A and A' are complimentary events, then

$$P(A) + P(A') = 1.$$

Theorem #13: If in an experiment the events A and B can both occur, then

$$P(A \cap B) = P(A)P(B|A).$$

Theorem #14: Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B).$$

Theorem #15: If in an experiment the events A_1, A_2, A_3,… A_k, can occur, then

$$P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_k) =$$

$$P(A_1)P(A_2|A_1)P(A_3|\ A_1 \cap A_2)\ldots P(A_k|\ A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_{k-1}).$$

If the events A_1, A_2, A_3,… A_k, are independent, then

$$P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_k) = P(A_1)P(A_2)P(A_3)\ldots P(A_k).$$

Theorem #16 (Theorem of Total Probability): If the events B_1, B_2,… B_k, constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1,2,\ldots,k$, then for any event A of S,

$$P(A) = \sum P(B_i \cap A) = \sum P(B_i)(A|B_i)$$

Theorem #17 (Bayes' Rule): If the events B_1, B_2,… B_k, constitute a partition of the sample space S, where $P(B_i) \neq 0$ for $i = 1,2,\ldots,k$, then for any event A of S, such that $P(A) \neq 0$,

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum P(B_i \cap A)}$$

$$= \frac{P(B_r)(A|B_r)}{\sum P(B_i)(A|B_i)} \quad \text{for } r = 1,2,\ldots,k$$

PART I. PROBABILITY TERMINOLOGIES & PROBLEMS

A. SAMPLE SPACE AND EVENTS

1. The set of all outcomes of a statistical experiment is called _____.
    A. Element
    B. Sample Space
    C. Sample Point
    D. Space
2. Each outcome in a sample space may be called three different names, which among the choices below does not refer to an outcome?
    A. Sample point
    B. Element
    C. Member
    D. Event
3. It is helpful to list the elements of a sample space systematically by means of branches referred to as ____.
    A. Diagram
    B. Branch and Bound Technique
    C. Tree Diagram
    D. Network diagram
4. Sample spaces with a large or infinite number of sample points are best described by a _____.
    A. Tree diagram
    B. Statement or Rule
    C. Flow diagram
    A. Network diagram
5. A/ An _____ is a subset of a sample space.
    A. Event
    B. Sample
    C. Element
    D. Member

6. The _____ of an event A with respect to S is the subset of all elements of S that are not in A.
    A. Intersection
    B. Union
    C. Complement
    D. Identity
7. The _____ of two events A and B, denoted by the symbol $A \cap B$, is the event containing all elements that are common to A and B.
    A. Intersection
    B. Union
    C. Complement
    D. Identity
8. Two events A and B are mutually exclusive or disjoint if $A \cap B = \varnothing$.
    A. True
    B. False
    C. Neither
    D. Not Sure,
9. The _____ of two events A and B, denoted by the symbol $A \cup B$, is the event containing all the elements that belong to A or B or both.
    A. Intersection
    B. Union
    C. Complement
    D. Identity
10. The relationship between events and the corresponding sample space can be illustrated by means of _____.
    A. Network diagram
    B. Tree diagram
    C. Flow diagram
    D. Venn diagram

11. Which of the following events are equal:

A = {1,3}

B = {x|x is a number on a die}

C = $\{x|x^2 - 4x + 3 = 0\}$

D = {x|x is the number of heads when six coins are tossed}

Note: B is the set of all x such that x is a number on a die.

D is the set of all x such that x is the number of heads….are tossed.

A. A & B
B. A & C
C. A & D
D. B & C

12. List the elements of the sample space, S = {x|x is a continent}.
    A. {N. America, S. America, Europe, Asia, Africa, Australia, Antartica}
    B. {America, Europe, Asia, Africa, Australia, Russia, Antartica}
    C. {America, Europe, Asia, Africa, Australia, Antartica, Japan}
    D. {N. America, S. America, Africa, Europe, Asia, Japan, Australia}
13. If S = {0,1,2,3,4,5,6,7,8,9} and A = {0,2,4,6,8}, B = {1,3,5,7,9}, C = {2,3,4,5}, D = {1,6,7}, list the elements of the sets corresponding to $A \cap B$.
    A. $A \cap B = \{0,1,2,3,4,5,6,7,8,9\}$
    B. $A \cap B = \{0,2,4,6,8\}$
    C. $A \cap B = \{1,3,5,7,9\}$
    D. $A \cap B = \varnothing$
14. Which of the following events are mutually exclusive?
    a. A golfer scoring the lowest 18-hole round in a 72-hole tournament and losing the tournament.
    b. A poker player getting a flush (all cards in the same suit) and 3 of a kind in the same 5-hand card.
    c. A mother giving birth to a baby girl and a set of twin daughters on the same day.
    d. A chess player losing the last game and winning the match.

A. A and B
B. A and C
C. B and D
D. B and C

B. COUNTING SAMPLE POINTS

15. If an experiment consists of throwing a die and then drawing a letter from the alphabet, how many points are in the sample space?
    A. S = 28 points
    B. S = 156 points
    C. S = 32 points
    D. S = 120 points
16. A drug for the relief of asthma can be purchased from 5 different manufacturers in liquid, tablet, or capsule form, all of which come in regular and extra strength. How many different ways can a doctor prescribe the drug for a patient suffering from asthma?
    A. 30 ways
    B. 15 ways
    C. 10 ways
    D. 8 ways
17. In how many different ways can a true-false test consisting of 10 questions be answered?
    A. 20 ways
    B. 10 ways
    C. 512 ways
    D. 1024 ways
18. In the CIE Certification exam, the Probability and Statistics part consists of 10 questions, each with 4 possible answers, of which only 1 is correct. In how many ways can an examinee check off one answer to each question and get all the answers wrong?
    A. 1,024 ways
    B. 59,049 ways

C. 512 ways
D. 1,048,576 ways

19. A witness to a hit-and-run accident told the police that the license number of the car contained the letters DRJ followed by 3 digits, the first of which is a 3. If the witness cannot recall the last 2 digits, but is certain that all three digits are different, find the maximum number of car registrations that the police may have to check.
    A. 56 registrations
    B. 72 registrations
    C. 90 registrations
    D. 19 registrations
20. It is an arrangement of all or part of a set of objects
    A. Combination
    B. Multiplication
    C. Permutation
    D. Circular permutation
21. It is the arrangement of objects in a circle.
    A. Combination
    B. Multiplication
    C. Permutation
    D. Circular permutation
22. Often, we are also concerned with the number of ways of partitioning a set of *n* objects into *r* subsets called ____.
    A. Sets
    B. Cells
    C. Ways
    D. Elements
23. The number of ways of selecting *r* objects from *n* without regard to order is called _____.
    A. Combinations
    B. Permutations
    C. Multiplicative Rule
    D. Circular permutation

24. In how many ways can 5 CIE Examinees be lined up to go inside the testing centers?
    A. 5 ways
    B. 120 ways
    C. 720 ways
    D. 60 ways
25. In how many ways can four dating reviewees be seated in the review center without restriction?
    A. 24 ways
    B. 384 ways
    C. 576 ways
    D. 4,320 ways
26. Find the number of ways that 6 lecturers in the CIE review be assigned to 4 classes if no lecturer is to be assigned to more than one class.
    A. 360 ways
    B. 24 ways
    C. 240 ways
    D. 36 ways
27. DRJ decided to give a raffle during the CIE review. Three tickets are drawn for first, second, and third prizes from a group of 40 CIE review students. Find the number of sample points in S for awarding the three prizes if each student holds only 1 ticket.
    A. 120
    B. 1,560
    C. 59,280
    D. 40,320
28. In how many ways can 6 students be seated in a round dining table?
    A. 120 ways
    B. 360 ways
    C. 720 ways
    D. 1,560 ways

29. In how many ways can CAM arrange his plants consisting of 4 orchids, 3 lemons, and 2 oreganos if one plant does not distinguish between plants of the same kind?
    A. 360 ways
    B. 720 ways
    C. 1,260 ways
    D. 1,560 ways
30. The UST Tigers plays 12 games during the UAAP season. In how many ways can the team end the season with 7 wins, 3 losses, and 2 ties?
    A. 1,260 ways
    B. 1,560 ways
    C. 5,040 ways
    D. 7,920 ways
31. How many ways are there to select 3 applicants from 8 equally qualified CIE's for a Capacity Planner position in a Semiconductor company.
    A. 24 ways
    B. 56 ways
    C. 120 ways
    D. 360 ways

C. PROBABILITY OF AN EVENT (ADDITIVE RULES)

32. If a letter is selected at random from the Alphabet, find the probability that the letter is listed somewhere after the letter J.
    A. 9/26
    B. 16/26
    C. 1/26
    D. 5/26
33. A car manufacturer is concerned about the possible recall of their best-selling pick-up trucks. If there was a recall, there is 0.25 probability that a defect is in the brake system, 0.18 in the transmission, 0.17 in the fuel system, and 0.40 in some other area. What is the probability that the defect is the brakes or the

fueling system if the probability of defects in both systems simultaneously is 0.15?

A. 0.27
B. 0.25
C. 0.18
D. 0.15

34. The probability that IEdeas will open a review center in Quezon City is 0.7, the probability that is will open a review center in Laguna is 0.4, and the probability that it will open a review center in either QC or Laguna or both is 0.8. What is the probability that IEdeas will locate in both cities?

A. 0.8
B. 0.3
C. 0.11
D. 0.4

35. What is the probability that IEdeas will locate in neither locations in problem # 34?

A. 0.2
B. 0.3
C. 0.4
D. 0.7

36. According to the recent survey conducted by HRAB, the probable location of LED TV screens in most hotels are: master bedroom – 0.40, kid's bedroom – 0.30, other bedroom – 0.10, other rooms – 0.20. What is the probability that an LED TV screen is located in a bedroom?

A. 0.40
B. 0.30
C. 0.20
D. 0.80

37. If the probabilities that a computer technician will repair 3,4,5,6,7, or 8 or more laptops on any given workday are 0.12, 0.19, 0.28, 0.24, 0.10, and 0.07, respectively, what is the probability that he will service at least 5 laptops on his next day at work?

A. 0.12

B. 0.19
C. 0.31
D. 0.69

D. CONDITIONAL PROBABILITY / MULTIPLICATIVE RULES

38. The probability that a husband watches Grace Anatomy is 0.40 and the probability that the wife watches the same show is 0.50. The probability that the husband watches the show given that his wife does is 0.70. Find the probability that the couple watches Grace Anatomy.
    A. 0.35
    B. 0.55
    C. 0.75
    D. 0.875
39. What is the probability that the wife watches the show given that her husband does?
    A. 0.35
    B. 0.55
    C. 0.75
    D. 0.875
40. What is the probability that at least 1 person from the married couple will watch the show.
    A. 0.35
    B. 0.55
    C. 0.75
    D. 0.875
41. Barangay Scout Fernandez has one fire truck and one ambulance available for emergencies. The probability that the fire truck is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. in the event that there is a fire in the barangay, find the probability that both the ambulance and the fire truck will be available.
    A. 0.9016
    B. 0.92

C. 0.96
D. 1.00

42. The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient enters a law suit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?
    A. 0.3
    B. 0.9
    C. 0.12
    D. 0.27
43. Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Find the probability that the event $A_1 \cap A_2 \cap A_3$ occurs, where A_1 is the event that the first card is a red two, A_2 the event that the second card is a jack or a queen, and A_3 is the event that the third card is greater than 5 but less than 9.
    A. 8/51
    B. 12/50
    C. 1/25
    D. 8/5525
44. The probability that a patient visiting his dentist will have an x-ray is 0.6; the probability that a patient who has an x-ray will also have a cavity filled is 0.3; and the probability that a patient who has had an x-ray and a cavity filled will also have a tooth extracted is 0.1. What is the probability that a patient visiting his dentist will have an x-ray, a cavity filled, and a tooth extracted?
    A. 0.018
    B. 0.10
    C. 0.30
    D. 0.60

E. BAYES' RULE

45. In a certain semiconductor plant, three machines, M_1, M_2, and M_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that these machines produce defective products at 2%, 3%, and

2%, respectively. Suppose that a finished product is randomly selected by the final QC Inspector, what is the probability that it is defective?

A. 0.006
B. 0.0245
C. 0.00135
D. 0.005

46. In problem #45, if a product were chosen randomly by the final QC Inspector, and found the product to be defective, what is the probability that it was made by machine M_3?

A. 0.0135
B. 0.0245
C. 0.2041
D. 0.2481

47. UST plans to enforce speed limits using radar traps at 4 different locations within the campus. The radar traps at each of the locations L_1, L_2, L_3, and L_4 are operated 40%, 30%, 20%, and 30% of the time, and if a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that he will receive a speeding ticket?

A. 0.27
B. 0.10
C. 0.08
D. 0.06

48. If in problem #47, the person received a speeding ticket on his way to work, what is the probability that he passed through the radar trap located at L_2?

A. 0.06
B. 0.08
C. 0.10
D. 0.11

49. Suppose that four Quality Control Inspectors at a canning factory are to stamp the expiration date on each package of canned good at the end of the assembly line. Aldrin, who stamps 20% of the packages, fails to stamp

the expiration date once in every 200 packages; JP, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Aaron, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Amathea who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages. If a customer complains that his package of canned goods does not show the expiration date, what is the probability that it was inspected by Aldrin?

A. 0.1124
B. 0.0089
C. 0.0070
D. 0.0077

50. A regional telephone company operates three identical relay stations at different locations. During a one year period, the number of malfunctions reported by each station and the causes are shown below:

STATION	**A**	**B**	**C**
Problems w/ electricity supplied	2	1	1
Computer malfunction	4	3	2
Malfunctioning electrical equipment	5	4	2
Caused by other human errors	7	7	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from Station C?

A. 0.1628
B. 0.1163
C. 0.2632
D. 0.4419

ANSWERS AND SOLUTIONS TO PROBLEMS

PART I.

1. B
2. D
3. C
4. B
5. A
6. C
7. A
8. A
9. B
10. D
11. B, $\{x|(x-1)(x-3)=0\} = \{1,3\}$
12. A
13. D
14. C
15. B → Using the Multiplication Rule: n1 = 6 and n2 = 26; n1 x n2 = 6 x 26 = 156 points
16. A →Using the Generalized Multiplication Rule: n1 = 5, n2 = 3, and n3 = 2 = n1 x n2 x n3 = 5 x 3 x 2 = 30 different ways.
17. D → Using the Generalized Multiplication Rule: n1 =2, n2 = 2,… n10 = 2; 2^{10} = 1056 ways to answer the test.
18. B → Using the Generalized Multiplication Rule: n1 =3, n2 = 3,… n10 = 3; 3^{10} = 59,049 ways to answer the test and get all the answers wrong!
19. B → Using the Multiplication Rule: n1 = 9 and n2 = 8; n1 x n2 = 9 x 8 = 72 car registrations.
20. C
21. D
22. B
23. A
24. B → Using theorem #3, n! = 5! = 5 x 4 x 3 x 2 x 1 = 120 ways

25. D → Using theorem #3, n! = 8! = 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 4,320 ways
26. A → Using theorem #4, ${}_nP_r = n! / (n - r)! = {}_6P_4 = 6! / (6 - 4)!$ = 360 ways
27. C → Using theorem #4, ${}_{40}P_3$ = 40! / (40 –*34*)! = 59,280 ways
28. A → Using theorem #5, (n-1)! = (6-1)! = 5! = 5 x 4 x 3 x 2 x 1 = 120 ways
29. C → Using theorem #6, $n! / n_1!n_2!...n_k!$. 9!/ 4!3!2! = 1,260 ways
30. D → Using theorem #7, $n! / n_1!n_2!...n_r!$. 12!/ 7!3!2! = 7,920 ways
31. B → Using theorem #8, ${}_nC_r = n! / r!(n- r) = {}_8C_3 = 8! / 3!(8 - 3)! = 56$ ways
32. B → Using theorem #9, P(A) = n / N = 16/26.
33. A → Using theorem #10, $P(A \cup B) = P(A) + P(B) – P(A \cap B)$

    Let P(A) – defect is in the brake system = 0.25

    P(B) – defect is in the fueling system = 0.17

    $P(A \cap B)$ – defect is in both systems = 0.15

    $P(A \cup B) = P(A) + P(B) – P(A \cap B) = 0.25 + 0.17 – 0.15 = 0.27$
34. B → Using theorem #10, $P(A \cup B) = P(A) + P(B) – P(A \cap B)$

    Let P(A) – IEdeas will locate in QC = 0.7

    P(B) – IEdeas will locate in Laguna = 0.4

    $P(A \cup B)$ – IEdeas will locate in either QC or Laguna = 0.8

    $P(A \cap B) = P(A) + P(B) – P(A \cup B) = 0.7 + 0.4 – 0.8 = 0.3$

35. A → Using theorem #12, $P(A' \cap B') = 1 – P(A \cup B) = 1 – 0.8 = 0.2$
36. D → $P(A_1 \cup A_2 \cup … \cup A_n) = P(A_1) + P(A_2) + … + P(A_n) = 0.40 + 0.30 + 0.10 = 0.80$
37. D → Using theorem #12, P(A) + P(A') = 1;

    Let E – event that at least 5 laptops are serviced

    P(E) = 1 – P(E')

    P(E) = 1- (0.12 + 0.19) = 0.69
38. A → Using theorem #13, $P(A \cap B) = P(A)P(B|A)$.

    Let H – husband watches Grace Anatomy

    W – wife watches the same show

    $P(W \cap H) = P(W)P(H|W) = (0.50)(0.70) = 0.35$

39. D → Using theorem #13, $P(W|H) = P(W \cap H) / P(H) = 0.35 / 0.4 = 0.875$

40. B → Using theorem #10, $P(W \cup H) = P(H) + P(W) - P(W \cap H) = 0.40 + 0.50 - 0.35 = 0.55$

41. A → Using theorem #14, $P(A \cap B) = P(A)P(B) = (0.98)(0.92) = 0.9016$

42. D → Using theorem #13, $P(A \cap B) = P(A)P(B|A)$.

Let A – the doctor makes a correct diagnosis

B – the patient sues

$P(A' \cap B) = P(A')P(B|A') = (0.30)(0.90) = 0.27$

43. D →Using theorem #15, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2|A_1)P(A_3| A_1 \cap A_2)$,

Let A_1 : the first card is a red 2

A_2: the second card is a jack or queen

A_3 : the third card is greater than 5 but less than 9

Since:

$P(A) = 2/52$; $P(A_2|A_1) = 8/51$ $P(A_3|A_1 \cap A_2) = 12/50$

$P(A_1 \cap A_2 \cap A_3) = (2/52)(8/51)(12/50) = 8/5525.$

44. A →Using theorem #15, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2|A_1)P(A_3| A_1 \cap A_2)$,

Let X : a patient has an x-ray

C: a cavity is filled

T : a tooth is extracted

Since:

$P(X) = 0.6$; $P(C|X) = 0.3$ $P(T|X \cap C) = 0.1$

$P(X \cap C \cap T) = (0.6)(0.3)(0.1) = 0.018.$

45. B → Using theorem # 16, $P(A) = \Sigma P(B_i \cap A) = \Sigma P(B_i)(A|B_i)$

Let A: product is defective

M_1: the product is made by machine M_1

M_2: the product is made by machine M_2

M_3: product is made by machine M_3

Applying the rule of elimination,

$P(A) = P(M_1)P(A|M_1) + P(M_2)P(A|M_2) + P(M_3)P(A|M_3)$.

$P(A) = (0.3)(0.02) + (0.45)(0.03) + (0.25)(0.02) = 0.0245$.

46. C → Using theorem #17,

$P(B_r|A) = P(B_r \cap A) / \Sigma P(B_i \cap A)$

$= P(B_r)(A|B_r) / \Sigma P(B_i)(A|B_i)$ for $r = 1,2,\ldots,k$

$P(M_3|A) = P(M_3)P(A|M_3) / [P(M_1)P(A|M_1) + P(M_2)P(A|M_2) + P(M_3)P(A|M_3)]$

$P(M_3|A) = (0.25)(0.02) / [(0.3)(0.02) + (0.45)(0.03) + (0.25)(0.02)]$

$P(M_3|A) = 0.2041$ therefore, this suggests that it was not made by M_3!

47. A → Using theorem #16,

Let: S_1,S_2, S_3, and S_4 represent the events that a person is speeding as he passes through the respective locations and R represent the event that the radar trap is operating resulting in a speeding ticket.

Probability that he will receive a speeding ticket, $P(S) = P(S_1)P(R|S_1) + P(S_2)P(R|S_2) + P(S_3)P(R|S_3) + P(S_4)P(R|S_4)$

$P(S) = (0.2)(0.4) + (0.1)(0.3) + (0.5)(0.2) + (0.2)(0.3) = 0.27$.

48. D → Using theorem #17,

$P(B_r|A) = P(B_r \cap A) / \Sigma P(B_i \cap A)$

$= P(B_r)(A|B_r) / \Sigma P(B_i)(A|B_i)$ for $r = 1,2,\ldots,k$

$P(S_2|R) = P(S_2)P(R|S_2) / [P(S_1)P(R|S_1) + P(S_2)P(R|S_2) + P(S_3)P(R|S_3) + P(S_4)P(R|S_4)] = 0.03 / 0.27 = 0.11.$

49.A → Use theorem #16 to solve for the total probability,

Let: A – no expiration date

B_1 – Aldrin is the inspector $P(B_1) = 0.20$ $P(A|B_1) = 0.005$ $= 0.0010$

B_2 – JP is the inspector $P(B_2) = 0.60$ $P(A|B_2) = 0.010 = 0.0060$

B_3 – Aaron is the inspector $P(B_3) = 0.15$ $P(A|B_3) = 0.011 = 0.0017$

B_4 – Amathea is the inspector $P(B_4) = 0.05$ $P(A|B_4) = 0.005$ $\underline{= 0.0002}$

$P(A) = 0.0089$

Using theorem #17, $P(B_1|A) = P(B_1)P(A|B_1) / P(A) = 0.0010/0.0089=0.1124.$

50. C → Use theorem #16 to solve for the total probability,

Let: A – malfunction by other human errors

B_1 – Station A $P(B_1) = 18/43$ $P(A|B_1) = 7/18$ $=(18/43)(7/18) = 0.1628$

B_2 – Station B $P(B_2) = 15/43$ $P(A|B_2) = 7/15 = (15/43)(7/15) = 0.1628$

B_3 – Station C $P(B_3) =10/43$ $P(A|B_3) =5/10 = (10/43)(5/10)$ $\underline{= 0.1163}$

$P(A) = 0.4419$

Using theorem #17, $P(B_3|A) = 0.1163 / 0.4419 = 0.2362.$

ASSESSMENT EXAM ON PROBABILITY

1. A pair of dice is tossed, find the probability of getting a total of 8.
    A. 8/36
    B. 5/36
    C. 12/36
    D. 5/18
2. Sir DRJ believes that under the present economic conditions, an investor will invest in tax-free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax-free bonds and mutual funds with a probability of 0.15. At this time, find the probability that an investor will invest in either tax-free bonds or mutual funds.
    A. 0.30
    B. 0.60
    C. 0.75
    D. 0.90
3. The probability that a car being filled with gasoline will also need an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.14. If the oil had to be changed, what is the probability that a new oil filter is needed?
    A. 0.25
    B. 0.35
    C. 0.40
    D. 0.56
4. A town has 2 fire engines operating independently. The probability that a specific engine is available when needed is 0.96. What is the probability that neither is available when needed?
    A. 0.04
    B. 0.08
    C. 0.0016
    D. 0.96
5. A paint store chain produces and sells latex and semigloss paint. Based on long-range sales, the probability that a customer will purchase latex is 0.75. of

those that purchase latex paint, 60% also purchase rollers. But 30%of semigloss buyers purchase rollers. Randomly selected buyer purchases a roller and a can of paint. What is the probability that the paint is latex?

A. 0.605
B. 0.751
C. 0.857
D. 0.955

6. The probabilities that a certain gas station will pump gas into 0,1,2,3,4, or 5 or more cars during a certain 30-minute period are 0.03, 0.18, 0.24, 0.28, 0.10, and 0.17, respectively. Find the probability that in this 30-minute period, more than 2 cars receive gas.

A. 0.73
B. 0.55
C. 0.42
D. 0.38

7. From a group of 5 men and 4 women, how many committees of size 3 are possible with no restrictions?

A. 84
B. 55
C. 40
D. 15

8. Three awards will be given for a batch of 25 CIE topnotchers. If each topnotcher can receive at most one award, how many possible selections are there?

A. 6,000
B. 12,000
C. 13,800
D. 15,000

9. Cai is going buy a computer for himself. He has a choice of buying either a laptop of a desktop. He also has a choice from 4 colors, 3 brands, and 5 suppliers. How many different ways can Cai choose to buy his computer?

A. 120
B. 60
C. 14
D. 2

10. How many sample points are in the sample space when a pair of dice is thrown once?
    A. 6
    B. 12
    C. 18
    D. 36

Answer and Solution to Exam:

1. B → Using theorem #9, N = n1n2 = 6x6 = 36; n = total of 8 = (2,6), (3,5), (4,4), (5,3), (6,2) = 5;
   P = n / N = 5/36.
2. C → Using theorem #10, $P(A \cup B) = P(A) + P(B) – P(A \cap B) =$
   0.6 + 0.3 – 0.15 = 0.75.
3. D → Using theorem #13, $P(A \cap B) = P(A)P(B|A)$
   Let C – oil change is needed
   F – oil filter is needed
   $P(F|C) = P(F \cap C) / P(C) = 0.14 / 0.25 = 0.56$
4. C → Using theorem #14, $P(A \cap B) = P(A)P(B)$
   Let A and B represent the availability of each fire engine
   $P(A' \cap B') = P(A')P(B') = (0.04)(0.04) = 0.0016$
5. C →Using theorem #16,
   Let A – customer purchases latex paint
   A' – customer purchases semigloss paint
   B – customer purchases rollers
   $P(A|B) = P(A)P(B|A) / P(A)P(B|A) + P(A')P(B|A')$
   $P(A|B) = (0.75)(0.60) / [(0.75)(0.60) + (0.25)(0.30)] = 0.857.$
6. B → Using theorem #12, P(A)+ P(A') = 1

$$P(X > 2) = 0.28 + 0.10 + 0.17 = 0.55$$

7. A → Using theorem #8, $n! / r!(n - r)!$.
   $9! / 3!(9-3)! = 84$

8. C → Using theorem #4, $nPr = n! / (n-r)! = {}_{25}P_3 = 25! / (25-3)!$
   $= (25)(24)(23) = 13{,}800$

9. A → Using theorem #2.2, $n_1 = 2$, $n_2 = 4$, $n_3 = 3$, $n_4 = 5$
   $(2)(4)(3)(5) = 120.$

10. D → Using theorem #1, $n_1 = 6$, $n_2 = 6 = n_1n_2 = (6)(6) = 36.$